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TEMPERATURE FIELD OF A SCREENED WALL WITH A THERMOACTIVE LINING ON EXPOSURE TO AN AXISYMMETRIC THERMAL EFFECT

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A mathematical model of the process of formation of a temperature field in the plane wall — thermoactive lining — heatproof coating system, the outer surface of which is under the influence of an axisymmetric heat flux with Gaussian-type intensity, is suggested as the base for developing a hierarchy of simplified analogs. To determine the temperature field studied, a finite integral transformation for a three-layer region with specific conjugation conditions has been developed.

Keywords: *screened wall, thermoactive lining, axisymmetric thermal effect, temperature field, integral transformation.*

Introduction. The necessity of solving the practically important problem of development of effective methods of thermal protection of constructions [1–3] is directly connected with the development of methods of mathematical simulation of the processes of formation of temperature fields in conjugated solid bodies [4–6]. In this case, usually a construction is imitated by an isotropic subspace or (much more rarely) by a plane wall that can be considered as an element of a real object of investigations.

A promising trend in the field of the thermal protection of structures is connected with the use of thermoelectric effects [7, 8], realizable by a thermoactive lining serving as a means of controlling the effect on the temperature field of an object subjected to external thermal effect [9]. The thermoactive lining forms an intermediate layer in the construction–heatproof coating system. In the simplest case it is a thin isotropic plate, on the outer surface of which a special film-type coating of negligibly small thickness is applied. As a result of controllable external effects, which can be connected with the regulated current strength, they absorb heat with an assigned specific power. This leads to specific conjugation conditions at the contact boundary that ensures the equality of temperatures and discontinuity of heat fluxes [9].

The difficulties in conducting a parametric analysis of temperature fields in multilayer regions are well known [4–6], just as different kinds of assumptions [9–11] leading to simplified analogs of the mathematical models used. The main aim of the investigations carried out is the development of the initial (base) mathematical model in the hierarchy of models of the heat-transfer process in a screened wall exposed to the action of an axisymmetric heat flux with Gaussian-type intensity in the presence of a thermoactive lining, as well as the use of this model to determine the corresponding temperature field.

Initial Model and Its Realizability. As an object of investigations, a screened wall with a thermoactive lining was used (Fig. 1). It was assumed that the unprotected wall surface is cooled by the surrounding medium, the temperature of which $T_{\text{s.m}}$ differs from the initial temperature T_{in} of the system under study, whereas the external surface of the heatproof coating is exposed to the action of an axisymmetric heat flux with Gaussian-type intensity. According to the main aim of the present investigations, in line with the initial assumptions and earlier published results [9], the initial mathematical model of the studied process of the formation of a temperature field can be represented in the following form:

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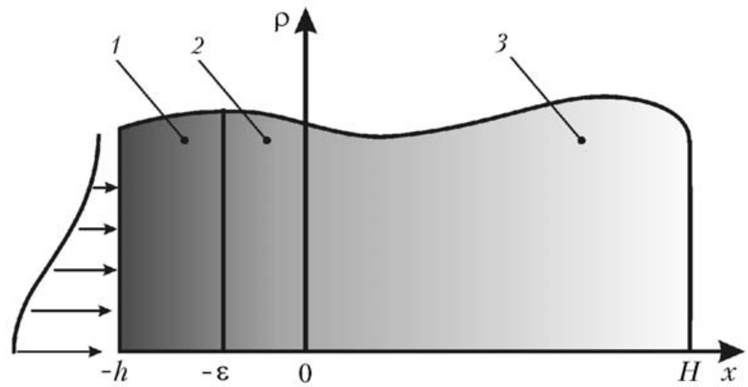

Fig. 1. Schematic diagram of the object of investigations: 1, heatproof coating; 2, thermoactive lining; 3, wall.

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = a(x)\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\theta}{\partial\rho}\right) + b(x)\frac{\partial^2\theta}{\partial x^2}, \quad \rho \ge 0, \quad -h < x < H, \quad \mathrm{Fo} > 0;$$

$$\theta(\rho, x, \mathrm{Fo})\big|_{\mathrm{Fo}=0} = 0; \quad \left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=-h} = -G\exp(-k^2\rho^2);$$

$$\theta(\rho, x, \mathrm{Fo})\big|_{x=-\varepsilon-0} = \theta(\rho, x, \mathrm{Fo})\big|_{x=-\varepsilon+0}; \tag{1}$$

$$\Lambda^-\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=-\varepsilon-0} - \left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=-\varepsilon+0} = -Q^-(\rho, \mathrm{Fo});$$

$$\theta(\rho, x, \mathrm{Fo})\big|_{x=0-0} = \theta(\rho, x, \mathrm{Fo})\big|_{x=0+0};$$

$$\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=0-0} - \Lambda^+\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=0+0} = -Q^+(\rho, \mathrm{Fo});$$

$$\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=H} = \gamma\left[J - \theta(\rho, x, \mathrm{Fo})\big|_{x=H}\right]; \quad \theta(\rho, x, \mathrm{Fo})\big|_{\substack{\mathrm{Fo}>0 \\ x\in(-h;H)}} \in L_\rho^2[0; +\infty),$$

where the piecewise-constant functionals are

$$a(x) = \begin{cases} a_c, & -h < x < -\varepsilon; \\ a_0, & -\varepsilon < x < 0; \\ 1, & 0 < x < H; \end{cases} \qquad b(x) = \begin{cases} b_c, & -h < x < -\varepsilon; \\ b_0, & -\varepsilon < x < 0; \\ 1, & 0 < x < H. \end{cases} \tag{2}$$

The last condition in (1) means that at each fixed values of $\mathrm{Fo} \ge 0$ and $x \in (-h; H)$ the function $\theta(\rho, x, \mathrm{Fo})$ is integrable with the square and weight ρ over the variable $\rho \in [0; \infty)$, i.e., is the inverse transform of the integral Hankel transformation of zero order [12]:

$$U(p, x, \mathrm{Fo}) = \mathrm{H}_0[\theta(\rho, x, \mathrm{Fo})] \equiv \int_0^\infty \theta(\rho, x, \mathrm{Fo})\,\rho J_0(p\rho)\,d\rho;$$

$$\theta(\rho, x, \mathrm{Fo}) \triangleq \mathrm{H}_0^{-1}[U(p, x, \mathrm{Fo})] \equiv \int_0^\infty U(p, x, \mathrm{Fo})\,pJ_0(p\rho)\,dp. \tag{3}$$

In this case, in Eq. (3) the equality should be understood as the equality over the norm in $L^2_\rho[0; +\infty)$ [12].

In the mathematical model (1), (2)

$$\rho = \frac{r}{l_0 + l_c}\,;\;\; x = \frac{z}{l_0 + l_c}\sqrt{\frac{\lambda_r^w}{\lambda_z^w}}\,;\;\; \mathrm{Fo} = \frac{t\lambda_r^w}{(l_0 + l_c)^2 c^w}\,;\;\; \theta = \frac{T - T_{in}}{T_{in}}\,;\;\; a_0 = \frac{\lambda_0 c^w}{\lambda_r^w c^0}\,;$$

$$a_c = \frac{\lambda_r^c c^w}{\lambda_r^w c^c}\,;\;\; b_0 = \frac{\lambda_0 c^w}{\lambda_z^w c^0}\,;\;\; b_c = \frac{\lambda_z^c c^w}{\lambda_z^w c^0}\,;\;\; \Lambda^- = \frac{\lambda_z^c}{\lambda^0}\,;\;\; \Lambda^+ = \frac{\lambda_z^w}{\lambda^0}\,;\;\; Q^- = W^- \frac{l_0 + l_c}{\lambda^0}\sqrt{\frac{\lambda_z^w}{\lambda_r^w}}\,;$$

$$Q^+ = W^+ \frac{l_0 + l_c}{\lambda^0}\sqrt{\frac{\lambda_z^w}{\lambda_r^w}}\,;\;\; h = \sqrt{\frac{\lambda_r^w}{\lambda_z^w}}\,;\;\; \varepsilon = \frac{l_0}{l_0 + l_c}\sqrt{\frac{\lambda_r^w}{\lambda_z^w}}\,;\;\; H = \frac{l_w}{l_0 + l_c}\sqrt{\frac{\lambda_r^w}{\lambda_z^w}}\,;\;\; J = \frac{T_{s.m} - T_{in}}{T_{in}}\,;$$

$$k^2 = k_*^2 (l_0 + l_c)^2\,;\;\; G = q\,\frac{l_0 + l_c}{\lambda_z^c}\sqrt{\frac{\lambda_z^w}{\lambda_r^w}}\,;\;\; \gamma = \alpha\,(l_0 + l_c)\sqrt{\frac{\lambda_z^w}{\lambda_r^w}}\,.$$

Next, we will consider the process of formation of the temperature field in the studied system (see Fig. 1) which from the outer side of the heatproof coating is subjected to the action of an axisymmetric flow with Gaussian-type intensity and from the outer surface of the wall is cooled by the surrounding medium of constant temperature. The process of heat redistribution in the system due to the difference between its initial temperature T_{in} and the surrounding medium temperature $T_{s.m}$ is not of interest to us, since by virtue of the linearity of the initial mathematical model this problem can be singled out and solved separately. Therefore in further considerations we formally assume that $T_{s.m} = T_{in}$, i.e., in (1) we assume that $J = 0$. Moreover, we will assume that at any fixed value $t \ge 0$ the specific surface powers of heat absorption $W^+(r, t)$, $W^-(r, t)$ of film coatings of the thermoactive lining are inverse transforms of the integral Hankel transformation of zero order (3).

The mathematical model (1), (2) represents a mixed problem for a system of partial differential equations of parabolic type and of the specific conditions of conjugation. With allowance for the assumptions made, for its solution we may use integral transformation (3) in the transforms of which the problem takes the following form [4, 12, 13]:

$$\frac{\partial U}{\partial \mathrm{Fo}} = b\,(x)\,\frac{\partial^2 U}{\partial x^2} - p^2 a\,(x)\,U\,,\quad -h < x < H\,,\quad \mathrm{Fo} > 0\,;\quad U\,(p, x, \mathrm{Fo})\big|_{\mathrm{Fo}=0} = 0\,;$$

$$\left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=-h} = -\frac{G}{2k^2}\exp\left(-\frac{p^2}{4k^2}\right);\quad U\,(p, x, \mathrm{Fo})\big|_{x=-\varepsilon-0} = U\,(p, x, \mathrm{Fo})\big|_{x=-\varepsilon+0}\,;$$

$$\Lambda^-\left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=-\varepsilon-0} - \left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=-\varepsilon+0} = -\,\Phi^-\,(p, \mathrm{Fo})\,; \tag{4}$$

$$U\,(p, x, \mathrm{Fo})\big|_{x=0-0} = U\,(p, x, \mathrm{Fo})\big|_{x=0+0}\,;$$

$$\left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=0-0} - \Lambda^+\left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=0+0} = -\,\Phi^+\,(p, \mathrm{Fo})\,;$$

$$\left.\frac{\partial U\,(p, x, \mathrm{Fo})}{\partial x}\right|_{x=H} + \gamma U\,(p, x, \mathrm{Fo})\big|_{x=H} = 0\,,$$

where

$$\Phi^-(p, \mathrm{Fo}) \triangleq \mathrm{H}_0\left[Q^-(\rho, \mathrm{Fo})\right]; \quad \Phi^+(p, \mathrm{Fo}) \triangleq \mathrm{H}_0\left[Q^+(\rho, \mathrm{Fo})\right]. \tag{5}$$

It is theoretically evident that for solving a mixed problem (2), (4), (5) defined by a system of parabolic-type partial differential equations, it is possible to apply the Laplace integral transformation for Fo [4, 5]. However, the problematic character of its transformation for multilayer regions is well known [4, 6, 9]. Therefore it is advisable to try to avoid these difficulties by developing a new integral transformation for a three-layer region with its subsequent application to the mixed problem (2), (4), (5) for the spatial variable $x \in (-h, H)$.

Integral Transformation for a Wall–Thermoactive Lining–Heatproof Coating Three-Layer Region. According to the general theory of integral transformations [12, 14], such a transformation is determined by a linear differential second-order operator:

$$L[\bullet] \equiv b(x)\frac{d^2 \bullet}{dx^2} - p^2 a(x) \bullet, \tag{6}$$

with boundary conditions at $x = -h$, $x = H$, and conjugation conditions at $x = -\varepsilon$, $x = 0$, which correspond to the conditions contained in mathematical model (1). In this case the functionals $a(x)$ and $b(x)$ are defined by equalities (2), whereas p is the parameter of the Hankel integral transformation of zero order (3). The weight function $P(x)$ that brings the linear differential operator $L[\cdot]$ to a regular form can be determined in a standard way [14]:

$$P(x) = \begin{cases} b_c^{-1}, & -h < x < -\varepsilon; \\ b_0^{-1}, & -\varepsilon < x < 0; \\ 1, & 0 < x < H. \end{cases} \tag{7}$$

If, in conformity with the general theory of integral transformations [12, 14], we require that the kernel $K(x, \mu)$ of the sought integral transformation be the solution of the corresponding problem:

$$\frac{d^2 K(x, \mu)}{dx^2} - p^2 a(x) P(x) K(x, \mu) = -\mu^2 P(x) K(x, \mu), \quad -h < x < H;$$

$$\left.\frac{dK(x, \mu)}{dx}\right|_{x=-h} = 0; \quad K(x, \mu)\big|_{x=-\varepsilon-0} = \Lambda^- K(x, \mu)\big|_{x=-\varepsilon+0};$$

$$\left.\frac{dK(x, \mu)}{dx}\right|_{x=-\varepsilon-0} = \left.\frac{dK(x, \mu)}{dx}\right|_{x=-\varepsilon+0}; \quad \Lambda^+ K(x, \mu)\big|_{x=0-0} = K(x, \mu)\big|_{x=0+0}; \tag{8}$$

$$\left.\frac{dK(x, \mu)}{dx}\right|_{x=0-0} = \left.\frac{dK(x, \mu)}{dx}\right|_{x=0+0}; \quad \left.\frac{dK(x, \mu)}{dx}\right|_{x=H} + \gamma K(x, \mu)\big|_{x=H} = 0,$$

where μ is the parameter of the sought-for integral transformation, then, according to (4)–(8), the following equality holds:

$$\int_{-h}^{H} L\left[U(p, x, \mathrm{Fo})\right] P(x) K(x, \mu)\, dx = -\mu^2 \int_{-h}^{H} U(p, x, \mathrm{Fo}) P(x) K(x, \mu)\, dx$$

$$-\Phi^+(p, \mathrm{Fo}) K(0-0, \mu) - \Phi^-(p, \mathrm{Fo}) K(-\varepsilon+0, \mu) + \frac{G}{2k^2}\exp\left(-\frac{p^2}{4k^2}\right) K(-h, \mu). \tag{9}$$

Moreover, assuming for the sake of definiteness that

$$0 < a_0 < a_c < 1 \,, \tag{10}$$

we see that for each fixed value of the parameter p of the Hankel integral transformation of zero order (3) there are three points on the $O\mu^2$ axis: $\mu^2 = a_0 p^2$, $\mu^2 = a_c p^2$, and $\mu^2 = p^2$ that determine the alternating sign intervals of the changes in the expressions

$$M_1 \triangleq \left\{\mu^2 - a(x)\,p^2\right\} P(x)\big|_{-h<x<-\varepsilon} = \frac{\mu^2 - a_c p^2}{b_c} \,,$$

$$M_2 \triangleq \left\{\mu^2 - a(x)\,p^2\right\} P(x)\big|_{-\varepsilon<x<0} = \frac{\mu^2 - a_0 p^2}{b_0} \,,$$

$$M_3 \triangleq \left\{\mu^2 - a(x)\,p^2\right\} P(x)\big|_{0<x<H} = \mu^2 - p^2 \,,$$

$$\mu^2 \in \left(0; a_0 p^2\right) \Rightarrow (M_1 < 0) \wedge (M_2 < 0) \wedge (M_3 < 0) \,, \tag{11}$$

$$\mu^2 \in \left(a_0 p^2; a_c p^2\right) \Rightarrow (M_1 < 0) \wedge (M_2 > 0) \wedge (M_3 < 0) \,,$$

$$\mu^2 \in \left(a_c p^2; p^2\right) \Rightarrow (M_1 > 0) \wedge (M_2 > 0) \wedge (M_3 < 0) \,,$$

$$\mu^2 \in \left(p^2; +\infty\right) \Rightarrow (M_1 > 0) \wedge (M_2 > 0) \wedge (M_3 > 0) \,.$$

Availing ourselves of Eqs. (2), (7), (8), and (11), we arrive at a set of equalities that determine the kernel $K(x, \mu)$ of the sought-for integral transformation:

$$K(x, \mu)\big|_{-h<x<-\varepsilon} = c_1 \varphi(x) \,, \quad \varphi(x) = \begin{cases} \cos\left[(x+h)\sqrt{M_1}\,\right], & \mu^2 \ge a_c p^2 \,; \\ \cosh\left[(x+h)\sqrt{-M_1}\,\right], & \mu^2 \le a_c p^2 \,; \end{cases}$$

$$K(x, \mu)\big|_{-\varepsilon<x<0} = c_2 \alpha(x) + c_3 \beta(x) \,,$$

$$\alpha(x) = \begin{cases} \cos\left(x\sqrt{M_2}\,\right), & \mu^2 \ge a_0 p^2 \,; \\ \cosh\left(x\sqrt{-M_2}\,\right), & \mu^2 \le a_0 p^2 \,; \end{cases} \quad \beta(x) = \begin{cases} \sin\left(x\sqrt{M_2}\,\right), & \mu^2 \ge a_0 p^2 \,; \\ \sinh\left(x\sqrt{-M_2}\,\right), & \mu^2 \le a_0 p^2 \,; \end{cases} \tag{12}$$

$$K(x, \mu)\big|_{0<x<H} = c_4 \psi(x) \,,$$

$$\psi(x) = \begin{cases} \gamma^{-1}\sqrt{M_3}\cos\left[(H-x)\sqrt{M_3}\,\right] + \sin\left[(H-x)\sqrt{M_3}\,\right], & \mu^2 \ge p^2 \,; \\ \gamma^{-1}\sqrt{-M_3}\cosh\left[(H-x)\sqrt{-M_3}\,\right] + \sinh\left[(H-x)\sqrt{-M_3}\,\right], & \mu^2 \le p^2 \,, \end{cases}$$

where in accordance with the conjugation conditions, presented in (8), the coefficients $\{c_j\}_{j=1}^4$ must satisfy the following homogeneous system of linear algebraic equations:

$$\varphi(-\varepsilon)\,c_1 - \Lambda^- \alpha(-\varepsilon)\,c_2 - \Lambda^- \beta(-\varepsilon)\,c_3 = 0 \,, \quad \varphi'(-\varepsilon)\,c_1 - \alpha'(-\varepsilon)\,c_2 - \beta'(-\varepsilon)\,c_3 = 0 \,,$$

$$\Lambda^+ \alpha(0)\,c_2 + \Lambda^+ \beta(0)\,c_3 - \psi(0)\,c_4 = 0 \,, \quad \alpha'(0)\,c_2 + \beta'(0)\,c_3 - \psi'(0)\,c_4 = 0 \,. \tag{13}$$

In this case, according to (11) and (12), the functionals $\varphi(x)$, $\psi(x)$, $\alpha(x)$, and $\beta(x)$ depend on the parameters of integral transformations p and μ:

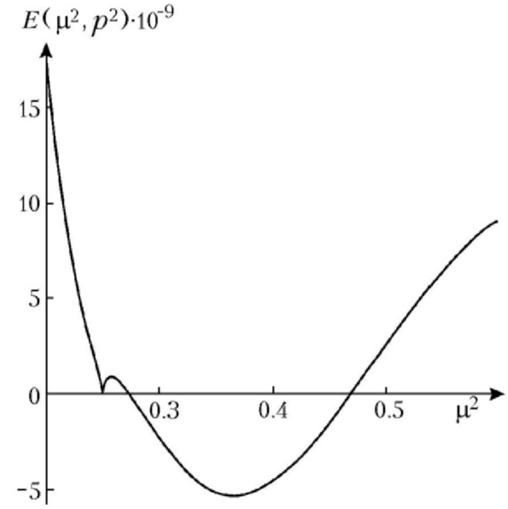

Fig. 2. Graphical representation of the solution of characteristic equation (17) at $p = 5$, $a_0 = 0.01$, $a_c = 0.02$.

$$\varphi(x) \equiv \varphi(x, \mu, p), \quad \psi(x) \equiv \psi(x, \mu, p), \quad \alpha(x) \equiv \alpha(x, \mu, p), \quad \beta(x) \equiv \beta(x, \mu, p) \tag{14}$$

and there are obvious identities

$$\alpha(0) \equiv 1, \quad \alpha'(0) \equiv 0 \equiv \beta(0). \tag{15}$$

The homogeneous system (13) has a nontrivial solution then and only then, when its determinant is equal to zero [15]. Thus, subject to (15), we have

$$E(\mu^2, p^2) \triangleq \begin{vmatrix} \varphi(-\varepsilon) & -\Lambda^{-}\alpha(-\varepsilon) & -\Lambda^{-}\beta(-\varepsilon) & 0 \\ \varphi'(-\varepsilon) & -\alpha'(-\varepsilon) & -\beta'(-\varepsilon) & 0 \\ 0 & -\Lambda^{+} & 0 & \psi(0) \\ 0 & 0 & -\beta'(0) & \psi'(0) \end{vmatrix} = 0. \tag{16}$$

Since from Eq. (14) the dependence of the functionals $\varphi(x)$, $\psi(x)$, $\alpha(x)$, and $\beta(x)$ on the parameters μ and p follows, Eq. (16) is the characteristic equation for finding the eigenvalues $\{\mu_k(p)\}_{k\geq 1}$ of the sought-for finite integral transformation at each fixed value of the parameter $p \in [0; +\infty)$ of the Hankel integral transformation of zero order (3) (Fig. 2). Having expanded the determinant on the left-hand side of Eq. (16) over the elements of the last column, we arrive at the equivalent representation of the characteristic equation, which is more convenient for practical use:

$$E(\mu^2, p^2) \equiv \beta'(0)\,\psi(0)\left\{\Lambda^{-}\alpha(-\varepsilon)\,\varphi'(-\varepsilon) - \alpha'(-\varepsilon)\,\varphi(-\varepsilon)\right\}$$

$$+ \Lambda^{+}\psi'(0)\left\{\Lambda^{-}\beta(-\varepsilon)\,\varphi'(-\varepsilon) - \beta'(-\varepsilon)\,\varphi(-\varepsilon)\right\} = 0. \tag{17}$$

If $\mu = \mu_k(p)$ is the root of characteristic equation (16) or of Eq. (17), equivalent to it, then, according to Eqs. (13), (15), and (17)

$$c_3 = d_3 c_4, \quad d_3 \equiv d_3(p, \mu) = \frac{\psi'(0)}{\beta'(0)}; \quad c_2 = d_2 c_4, \quad d_2 \equiv d_2(p, \mu) = \frac{\psi(0)}{\Lambda^{+}};$$

$$c_1 = d_1 c_4, \quad d_1 \equiv d_1(p, \mu) = \frac{\alpha'(-\varepsilon)\,\beta'(0)\,\psi(0) + \Lambda^{+}\beta'(-\varepsilon)\,\psi'(0)}{\Lambda^{+}\psi'(-\varepsilon)\,\beta'(0)} \tag{18}$$

$$\equiv \frac{\Lambda^{-}\left[\alpha(-\varepsilon)\beta'(0)\psi(0)+\Lambda^{+}\beta(-\varepsilon)\psi'(0)\right]}{\Lambda^{+}\varphi(-\varepsilon)\beta'(0)} .$$

Thus, Eqs. (12) and (18) yield

$$K(x,\mu) = c_4 \begin{cases} d_1\varphi(x), & -h < x < -\varepsilon ; \\ d_2\alpha(x) + d_3\beta(x), & -\varepsilon < x < 0 ; \\ \psi(x), & 0 < x < H , \end{cases} \tag{19}$$

where the functional $c_4 \equiv c_4(p, \mu)$ is determined from the condition of normalization of the kernel of the sought-for integral transformation:

$$1 = \left\|K(x,\mu)\right\|^2 = \int_{-h}^{H} P(x)\, K^2(x,\mu)\, dx .$$

Since the weight function was defined by equality (7) and the representation (19) holds, then, subject to (14) and (18), we find

$$c_4(p,\mu) = \left\{ \frac{d_1^2(p,\mu)}{b_c} \int_{-h}^{-\varepsilon} \varphi^2(x,\mu,p)\, dx \right.$$

$$\left. + \frac{1}{b_0} \int_{-\varepsilon}^{0} \left[d_2(p,\mu)\,\alpha(x,\mu,p) + d_3(p,\mu)\,\beta(x,\mu,p)\right]^2 dx + \int_0^H \psi^2(x,\mu,p)\, dx \right\}^{-1/2} , \tag{20}$$

and the finite integral transformation over the spatial variable $x \in [-h; H)$

$$V_k(p,\mathrm{Fo}) \triangleq \int_{-h}^{H} U(p,x,\mathrm{Fo})\, P(x)\, K(x,\mu_k)\, dx ; \quad U(p,x,\mathrm{Fo}) = \sum_{k=1}^{\infty} V_k(p,\mathrm{Fo})\, K(x,\mu_k) \tag{21}$$

is entirely determined by equalities (7), (11), (12), (18)–(20) and by characteristic equation (17). The equality in (21) is understood as the equality over the norm in the linear space $L_P^2[-h; H]$, and all the integrals on the right-hand side of equality (20) are calculated in quadratures [16].

Temperature Field. In the transforms of the finite integral transformation (21) applied for the spatial variable $x \in [-h; H]$, subject to (6) and (9), for each fixed value $k \in \{1, 2, ...\}$. a mixed problem (4) for a system of partial differential equations of parabolic type represents a Cauchy problem for determining the functional $V_k(p, \mathrm{Fo})$:

$$\frac{dV_k(p,\mathrm{Fo})}{d\mathrm{Fo}} = -\mu_k^2 V_k(p,\mathrm{Fo}) - \Phi^{+}(p,\mathrm{Fo})\, K(0-0,\mu_k) - \Phi^{-}(p,\mathrm{Fo})\, K(-\varepsilon+0,\mu_k)$$

$$+ \frac{G}{2k^2} K(-h,\mu_k) \exp\left(-\frac{p^2}{4k^2}\right), \quad \mathrm{Fo} > 0 ; \quad V_k(p,0) = 0 ,$$

whose solution can be found by standard methods [17]:

$$V_k(p,\mathrm{Fo}) = \frac{G}{2k^2\mu_k^2} K(-h,\mu_k) \exp\left(-\frac{p^2}{4k^2}\right)$$

$$-\int_0^{\text{Fo}} \left\{ \Phi^+ (p, s)\, K(0-0, \mu_k) - \Phi^- (p, s)\, K(-\varepsilon + 0, \mu_k) \right\} \exp\left[-\mu_k^2 (\text{Fo} - s) \right] ds\,. \quad (22)$$

To determine the temperature field $\theta(\rho, x, \text{Fo})$ of the wall-thermoactive lining–heatproof coating system studied, it is sufficient to make use of the representation (22) for the functional $V_k(p, \text{Fo})$, having first employed the formula of inversion of the finite integral transformation (21), i.e., to go over to the transforms $U(\rho, x, \text{Fo})$, and thereafter convert the Hankel integral transformation of zero order (3).

CONCLUSIONS

1. The main difficulties connected with the practical use of the finite integral transformation (21) for a three-layer region are due to the dependence of its kernel on the parameter p of the Hankel integral transformation of zero order applied at the preceding stage of the solution of initial problem. This leads, in particular, to the necessity of determining the spectrum of the eigenvalues of the finite integral transformation (21) in the form of the parametric family $\{\mu_k(p)\}_{k\geq 1}^{\infty}$.

2. The dependence of the kernel of the finite integral transformation (21) on the parameter of the integral transformation applied at the preceding stage is affected not only by the lamellar nature of the structure of the region where the process studied proceeds, but also by the dimensionality of the problem being solved. In the absence of one of these two factors the integral transformation is free of the indicated drawback, which directly follows from the structure of the operator $L[\cdot]$ which is determined by identity (6).

3. The representation of the temperature field obtained by using the developed finite integral transformation does not allow one to effectively carry out a parametric analysis and to solve the problems of controlling the temperature field of the system studied, simulated by the plane wall-thermoactive lining-heatproof coating construction.

4. To ensure the possibility of carrying out a parametric analysis of the processes of heat transfer in the system studied and solving the problems of controlling its temperature field it is necessary, by using the initial mathematical model (1), (2) and a base one, to develop a hierarchy of simplified analogs and determine the region of admissible application of each of them.

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NOTATION

c, volumetric specific heat, J/(m^3·K); Fo, Fourier number; H, dimensionless wall thickness; $H_0[\cdot]$, $H_0^{-1}[\cdot]$, operators of direct and inverse Hankel integral transformations of zero order; h, dimensionless thickness of heat shielding coating; $J_0(\cdot)$, Bessel function of first kind and zero order; $K(\cdot)$, kernel of developed integral transformation; $L[\cdot]$, linear differential operator; $L_\rho^2[0; +\infty)$, linear space of the functions integrated with a square and weight ρ on semilimited interval $[0; +\infty\}$; $L_P^2[-h, H]$, linear space of functions integrated with a square over the interval $[-h, H]$; l_{w}, wall thickness, m; l_0, thickness of thermoactive lining, m; l_{c}, thickness of heatproof coating, m; k_*, coefficient of focusing of external heat flux, 1/m; $P(\cdot)$, weight function of the integral transformation developed; p, parameter of Hankel integral transformation of zero order; q, density (intensity) of a heat flux, W/m^2; r, radius, m; T, temperature, K; t, time, sec; W, specific (per unit area) power of heat absorption of the film coating of thermoactive lining, W/m^2; x, dimensionless spatial variable; z, spatial variable, m; α, heat transfer coefficient, W/(m^2·K); ε, dimensionless thickness of thermoactive lining; θ, dimensionless temperature; λ, thermal conductivity, W/(m·K); μ, parameter of developed integral transformation; ρ, dimensionless radius. Subscripts and superscripts: c, heatproof coating; in, initial value; s.m, surrounding medium; w, wall; "–" and "+," surfaces, $z = -l_0$ and $z = 0$, of the thermoactive lining, respectively; 0, thermoactive lining.

REFERENCES

1. Yu. V. Polezhaev and F. B. Yurevich, *Thermal Shielding* [in Russian], Énergiya, Moscow (1976).
2. V. S. Zarubin, *Calculation and Optimization of Thermal Insulation* [in Russian], Énergoatomizdat, Moscow (1991).
3. Yu. V. Polezhaev, S. V. Reznik, É. B. Vasilevskii, et al., *Materials and Coatings under Extremal Conditions. Outlook on the Future*, in 3 vols., Vol. 1, *Prediction and Analysis of Extremal Effects* [in Russian], Izd. N. É. Bauman MGTU, Moscow (2002).
4. A. V. Luikov, *Heat Conduction Theory* [in Russian], Vysshaya Shkola, Moscow (1967).
5. É. M. Kartashov, *Analytical Methods in the Theory of Thermal Conductivity of Solid Bodies* [in Russian], Vysshaya Shkola, Moscow (2001).
6. V. A. Kudinov, É. M. Kartashov, and V. V. Kalashnikov, *Analytical Solutions of the Problems of Heat and Mass Transfer and Thermoelasticity for Multilayer Constructions* [in Russian], Vysshaya Shkola, Moscow (2005).
7. E. I. Kim, V. T. Omel'chenko, and S. N. Kharin, *Mathematical Models of Thermal Processes in Electric Contacts* [in Russian], Nauka, Alma-Ata (1977).
8. L. I. Anatychuk, *Thermoelements and Thermoelectric Devices* [in Russian], Naukova Dumka, Kiev (1979).
9. A. V. Attetkov, I. K. Volkov, and E. S. Tverskaya, Thermoactive spacer as a means of controllable influence on the temperature field of a construction, *Izv. Ross. Akad. Nauk*, *Énergetika*, No. 4, 131–141 (2002).
10. A. V. Attetkov, I. K. Volkov, and E. S. Tverskaya, Mathematical simulation of a temperature field in a screened half-space with a thermoactive spacer, in: *Proc. 4th Russian National Heat Transfer Conf.*, Vol. 7, *Radiative and Combined Heat Transfer. Heat Conduction*, *Heat Insulation* [in Russian], Moscow (2006), pp. 156–159.
11. A. V. Attetkov, I. K. Volkov, and E. S. Tverskaya, Mathematical modeling of the process of heat transfer in a shielded half-space with a thermoactive spacer under external thermal action, *Inzh.-Fiz. Zh.*, **81**, No. 3, 559–568 (2008).
12. M. A. Naimark, *Linear Differential Operators* [Russian translation], Nauka, Moscow (1969).
13. H. Bateman and A. Érdeyi, *Tables of Integral Transforms*, in 2 vols., Vol. 2, *Bessel Transformation*, *Integrals of Special Functions* [Russian translation], Nauka, Moscow (1970).
14. N. S. Koshlyakov, É. B. Gliner, and M. M. Smirnov, *Partial Derivatives Equations of Mathematical Physics* [in Russian], Vysshaya Shkola, Moscow (1970).
15. G. E. Shilov, *Mathematical Analysis* (*Finite- Dimensional Linear Spaces*) [in Russian], Nauka, Moscow (1969).
16. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals*, *Sums*, *Series*, *and Products* [in Russian], Nauka, Moscow (1971).
17. L. É. Él'sgol'ts, *Differential Equations and Variational Calculus* [in Russian], Nauka, Moscow (1969).